Hadronic Form Factors: Combining QCD Calculations with Analyticity

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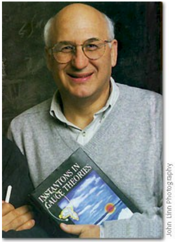
Foreword



"Unlike some models whose relation to Nature is still a big question mark, QCD will stay with us forever."

in " Persistent Challenges of QCD", Lilienfeld Prize Lecture, (2006)

- QCD $\Rightarrow$ hadrons   is there an analytic solution ?
- approximate methods: perturbative QCD, lattice QCD, effective theories,...
- QCD sum rules, 30 years old, still a very important tool
  "QCD and Resonance Physics. Sum Rules. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Published in Nucl.Phys.B147:385-447,1979. | Cited 3516 times
- Misha's interest in "Squaring the amplitudes"
- Light-cone sum rules: an "offspring" of QCD sum rules:

Hadronic form factors

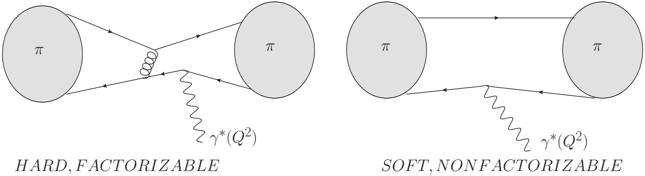
- the simplest hadronic transition amplitudes:

$$\langle \pi(p+q)|j_\mu^{em}|\pi(p)\rangle = (2p+q)_\mu F_\pi(Q^2) \qquad Q^2 = -q^2$$

$$\langle \pi(p)|\bar{q}\gamma_\mu b|B(p+q)\rangle = (2p+q)_\mu f^+_{B\pi}(q^2) + ..., \qquad B \to D$$

- pointlike quark transition, large scale (Q^2 , m_b, m_c)
  $\Rightarrow$ interplay of hard and soft quark-gluon interactions
- $F_\pi(Q^2)$: a popular testing ground of AdS/QCD
- Applications to flavour physics:
  - $B \to \pi$ form factor needed to extract $|V_{ub}|$ from data on semileptonic decay rate $B \to \pi l \nu_l$
  - $D \to \pi, K$ form factors $\Rightarrow$ $|V_{cd}|,|V_{cs}|$

Pion electromagnetic form factor



- QCD asympotitics, a convolution:

$$F_\pi(Q^2)^{asympt} = \frac{8\pi\alpha_s f_\pi^2}{9Q^2}\left(\int_0^1 du \frac{\varphi_\pi(u,\mu)}{\bar{u}}\right)^2\Bigg|_{\mu\sim Q},$$

- universal pion distribution amplitude :
  vacuum-pion matrix element expanded near $x^2 = 0$

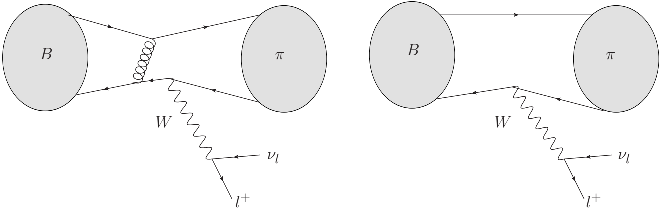
$$\langle\pi(p)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -ip_\mu f_\pi \int_0^1 du\, e^{iup\cdot x}\varphi_\pi(u)$$

[Chernyak, Zhitnisky; Efremov,Radyushkin; Brodsky-Lepage (1977-1980)]

- how large is the “soft” part ? $\sim 1/Q^4$

Heavy-light Form factor

- The factorization pattern



$HARD, FACTORIZABLE$ $SOFT, NONFACTORIZABLE$

$$f_{B\pi}(q^2) = \alpha_s(\mu) \int d\omega du \phi_+^B(\omega,\mu) T_h(q^2,\omega,u,\mu)\varphi_\pi(u,\mu)|_{\mu\sim m_b} + f_{B\pi}^{soft}(q^2)$$

T_h-hard-scattering amplitude, $\varphi_\pi(u)$ -light-cone DA of pion
φ_B -light-cone DA of B (defined in HQET)

- how large is $f_{B\pi}^{soft}(q^2)$?, no suppression in $1/m_B$

Light-cone sum rules (LCSR)

[I.Balitsky,V.Braun et al (1989); V.Chernyak, I.Zhitnitsky (1989)]

correlator = dispersion relation $\oplus$ unitarity $\Rightarrow$ form factors

- $B \to \pi$: correlator of $j^W_\mu = \bar{u}\gamma_\mu b$ and $j_5 = m_b \bar{b} i\gamma_5 d$

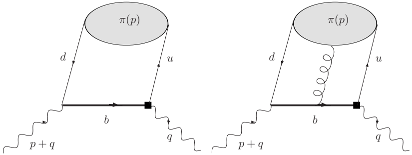
$$\int d^4x e^{iqx} \langle \pi(p)| T\{j_W(x) j_5(0)\}|0\rangle = \frac{\langle \pi | j_W | B\rangle \langle B | j_5 | 0\rangle}{m_B^2 - (p+q)^2} + \sum_h \frac{\langle \pi | j_W | h\rangle \langle h | j_5 | 0\rangle}{m_h^2 - (p+q)^2}$$

$$q^2, (p+q)^2 \ll m_b^2 \Downarrow \quad x^2 \to 0$$

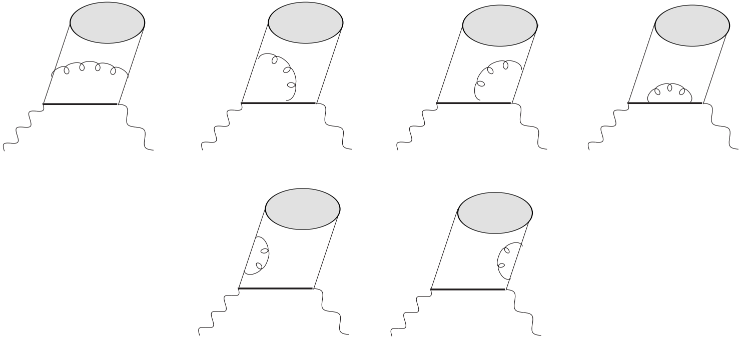
$$\sum_{t=2,3,..} C_t(x^2, m_b) \langle \pi(p)| O_t(x,0)|0\rangle \Leftarrow \text{OPE near LC}$$

- $\langle \pi(p)| O_t(x,0)|0\rangle$ - pion light-cone distribution amplitudes, twist /Fock-state expansion:
  $O_2 = \bar{u}(x)\gamma_\mu\gamma_5 d(0) \Rightarrow \varphi_\pi(u) \oplus tw4$, $O_3 = \bar{u}(x) i\gamma_5 d(0) \Rightarrow \varphi_{3\pi}(u)$,
  $\widetilde{O}_3 = \bar{u}(x) G_{\mu\nu}(vx)\gamma_5 d(0) \Rightarrow \Phi_{3\pi}(\alpha_1, \alpha_2)$, ...

Diagrams of the OPE



$\Leftarrow$ LO , including 3-particle DA's



NLO, collinear factorization:

[A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997)]

Practical use of LCSR

- matching OPE and dispersion relation at
  $|(p+q)^2| \sim m_b\chi \to M^2$ and using quark-hadron duality:

$$F_{OPE}((p+q)^2, q^2) = \sum_{t=2,3,4} \int du\, C_t((p+q)^2, q^2, u) \otimes \varphi_\pi^t(u)$$

$$= \frac{m_B^2 f_B f_{B\pi}^+(q^2)}{m_B^2 - (p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s, q^2)]_{OPE}}{s-(p+q)^2}$$

- inputs on l.h.s.: $\overline{m}_b$, α_s, $\varphi_\pi^{(t)}(u)$, t=2,3,4
- f_B- measured $(\times|V_{ub}|)$
  and/or determined from QCD (SVZ) sum rule
- uncertainties due to:
  - variation of input parameters, scales
  - quark-hadron duality, s_0^B

(controlled by the m_B calculation)

Universality of the method

- LCSR contains *both* “soft” and “hard” contributions to $f_{B\pi}$:
  the soft contribution dominates (no α_s!)
- the calculation is done at finite m_b,
  no $m_Q \to \infty$ limit involved
- heavy-quark expansion of LCSR $\Rightarrow$ $f_{B\pi} \sim 1/m_b^{3/2}$
  (Λ_{QCD}/m_b) and/or (Λ_{QCD}/χ) suppression of higher twists
- switching $b \to c$ in the correlator
  $\Rightarrow$ LCSR for $D \to \pi, K$ form factors
- $b \to u, d$, at $|(p+q)^2|, Q^2 = -q^2 \gg \Lambda_{QCD}$
  $\Rightarrow$ LCSR for the pion form factor $F_\pi(Q^2)$
  [V. Braun, I. Halperin(1997)]; [V. Braun, A.K., M.Maul (2000)],
  [J.Bijnens, A.K. (2002)]
  reproduces the QCD asymptotics $\sim \alpha_s/Q^2$ at $Q^2 \to \infty$

Limitations of the method

- only the correlator is calculable,
  no "direct access" to the hadronic matrix element
  (the same in lattice QCD)
- accuracy (estimated at $\sim \pm 15\%$ level) limited by:
  - uncertainty of $\varphi_\pi(u)$ (Gegenbauer moments)
  - truncated twist expansion
  - accuracy of the quark-hadron duality approximaiton

  [M. Shifman (2001)]
- the region of accessible q^2 is restricted:

  $f^+_{B\pi}(q^2)$ at $q^2 \ll (m_B - m_\pi)^2$, $f^+_{D\pi}(q^2)$ at $q^2 = 0$,
  $F_\pi(Q^2)$ at $Q^2 = -q^2 \geq 1$ GeV^2
- Can we access other regions of q^2?
  where more data are available

Employing the analyticity of the form factors

- Form factors are analytic functions of q^2
  $\Rightarrow$ dispersion relation, e.g.:

$$f^+_{B\pi}(q^2) = \frac{f_{B^*} g_{B^* B\pi}}{m^2_{B^*} - q^2} + \frac{1}{\pi} \int\limits_{(m_B+m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} f^+_{B\pi}(s)}{s - q^2}$$

(no subtractions due to the QCD asymptotics)

- $f^+_{B\pi}(q^2)$ real at $q^2 < m^2_{B^*}$, pole at $q^2 = m^2_{B^*}$, branch points (and poles) at $q^2 > (m_B + m_\pi)^2$
- dispersion relation valid everywhere:
  match it to LCSR at $q^2 \ll m^2_B$ and analytically continue
- but: the integral can only be treated in a model,
  e.g., an effective pole

Conformal mapping

- map the complex q^2-plane onto $|z| < 1$ in the z-plane:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_\pi), t_0 < t_+$$

[N. Meiman (1963)]; [B.Ioffe, B.Geshkenbein (1963)], [S.Okubo (1971)]

- many applications to $B \to \pi$ and other form factors:
  [C.G.Boyd, B.Grinstein, R.Lebed (1995)],
  [L.Lellouch (1996)],..., [T.Becher, R.Hill (2006)],..
- combined with perturbative QCD bounds
  (from the unitarity for the 2-point correlation function)
- bounds not that much restrictive, $|z_{max}| \leq 0.3$, $\Rightarrow$
  a simple Taylor expansion near $z = 0$ suffices

Series paramaterizations

- The last (and simplest) version
  [C. Bourrely, I. Caprini, L. Lellouch (2008)]
  a power expansion with $\sim 3, 4$ parameters

$$f^+_{B\pi}(q^2) = \frac{1}{1 - q^2/m^2_{B^*}} \sum_{k=0}^{k_{max}} a_k \Big(z(q^2, t_0) \Big)^k$$

  B^* the vector-meson pole near threshold

- the recipe:
  fit a_k to a $[ff]_{LCSR}$ in the "trusted "region of $q^2 \Rightarrow z$,
  continue over $z \Rightarrow q^2$ beyond that region
  (caution: stay below threshold !)

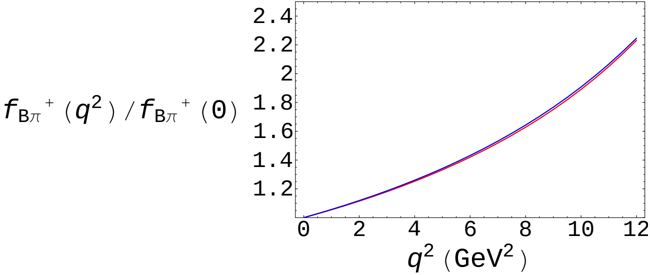
$B \to \pi$ form factor, result from LCSR

- recent update at $q^2 = 0$,

  [G.Duplancic, A.K., B.Melic, Th.Mannel, N.Offen (2007)

  $$f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}$$

- with f_B obtained from two-point QCD sum rules
- fitting the slope to the BaBar data:

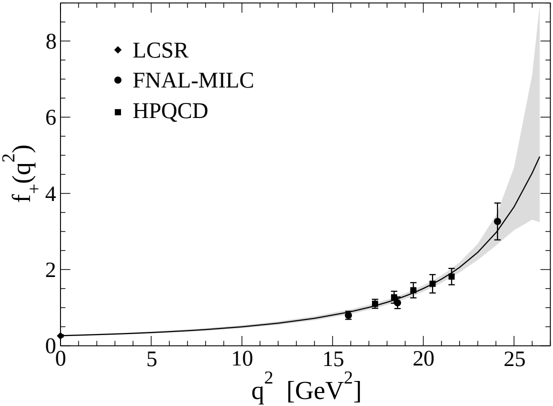


to fix the parameters of φ_π (Gegenbauer moments):

$$\varphi_\pi(u, 1\text{GeV}) =$$
$$6u(1-u)\Big(1+[0.16\pm0.01]C_2^{3/2}(2u-1)+[0.04\pm0.01]C_4^{3/2}(2u-1)\Big)$$

Current status of $B \to \pi$ form factor

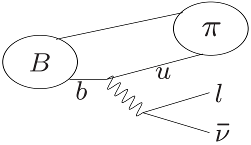
from [Bourrely, Caprini, Lellouch, 0807.222 hep-ph]



$q^2 = 0$: LCSR *[this work]* , $q^2 >$15 GeV^2: lattice QCD *[FNAL-MILC, HPQCD]*
the curve: series (in z) parameterization

"Squaring the amplitude"

- semileptonic decay: $\overline{B}_d \to \pi^+ l \overline{\nu}_l$



$\Rightarrow |V_{ub}|$

$$d\Gamma(\overline{B}_d \to \pi^+ l \bar{\nu}_l)/dq^2 = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2$$

semileptonic region: $0 < q^2 < (m_B - m_\pi)^2 = q^2_{max} \simeq 26.4$ GeV^2

- accurately measured BR's and slopes,
  $BR(\bar{B}^0 \to \pi^- l \bar{\nu}) = (1.36 \pm 0.09) \times 10^{-4}$ [PDG '08]
  we use $dBR/dq^2|_{q^2=0}$

Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

[ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
Okamoto et al. '05	lattice ($n_f = 3$)	-	3.78±0.25±0.52
HPQCD '06	lattice ($n_f = 3$)	-	3.55±0.25±0.50
Flynn et al '07	-	lattice $\oplus$ LCSR	$3.47 \pm 0.29 \pm 0.03$
Ball, Zwicky '04	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
DKMMO '07	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$
Bourrely, Caprini, Lellouch '08	-	lattice$\oplus$ LCSR	3.54 ± 0.24

- agrees well with the unitarity triangle fit

[see CKM fitter group web site]

$D \to \pi, K$ form factors from LCSR

[Ch.Klein, A.K., Th. Mannel, N.Offen, paper in prepar.]

- replace $b \to c$, $u \to d(s) \Rightarrow$ LCSR for $D \to \pi(K)$
- LCSR predicts the product, $[f_D f_{D\pi}(0)]_{LCSR} = 138^{+20}_{-13}$ MeV
- exp.data *[CLEO '08]* :

$$BR(D \to l\nu_l) \Rightarrow f_D|V_{cd}| = 46.5 \pm 2.0 \text{ MeV}$$

$$dBR/dq^2(D \to \pi l\nu_l) \Rightarrow f_{D\pi}(0)|V_{cd}| = 0.143 \pm 0.005 \pm 0.002,$$

- multiply two exp. numbers and divide by LCSR prediction:

$$|V_{cd}| = 0.219 \pm [0.005]_{exp1} \pm [0.004]_{exp2}{}^{+0.016}_{-0.010},$$

the LCSR error is effectively halved !

CLEO$\oplus$ lattice: $|V_{cd}| = 0.223 \pm 0.008 \pm 0.003 \pm 0.023$

Determination of $|V_{cs}|$

- from the predicted ratio of $D \to \pi$ and $D \to K$ form factors and CLEO data (f_D cancels):

$$\frac{|V_{cd}|}{|V_{cs}|} = 0.214 \pm [0.008]_{exp} \pm [0.002]_{exp} \pm 0.014\,,$$

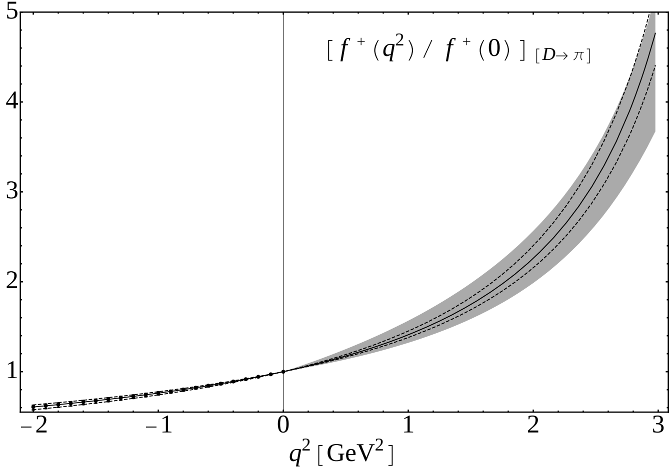
$$|V_{cs}| = 1.03 \pm [0.08]_{ratio} \begin{bmatrix} +0.08 \\ -0.06 \end{bmatrix}_{V_{cd}}\,,$$

- compare with CLEO$\oplus$ lattice:
  $|V_{cs}| = 1.019 \pm 0.019 \pm 0.007 \pm 0.106$

$D \to \pi$ form factor shape [preliminary]

points -LCSR at $q^2 \leq 0$

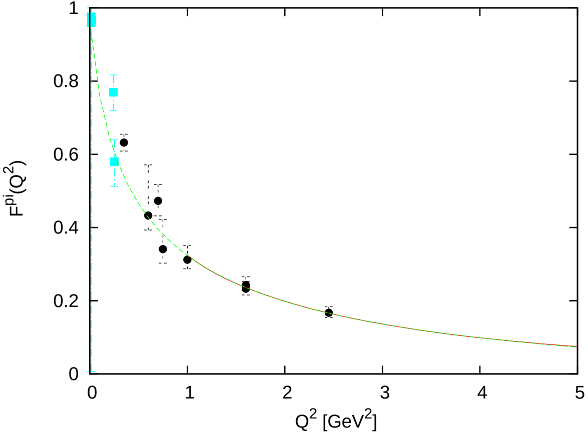
solid (dashed) fit to z-expansion $\to q^2 > 0$ (uncert.)



shaded-CLEO data

The pion e.m. form factor from LCSR [preliminary]

- LCSR *[V.Braun, A.K., M.Maul (2000); J.Bijnens, A.K.(2001)]* recalculated at $Q^2 = 1.0 - 5.0$ GeV^2 (solid), fit to the series (z)-param. (dashed) $\to Q^2 < 1.0$ GeV^2 with the same pion DA as for $B \to \pi$
- points Jlab data [2008], points *[FNAL, Amendolia et al (1986)]*



Summary

- Approximate method of QCD light-cone sum rules $\oplus$ analyticity (series parameterization) works for $B \to \pi$, $D \to \pi, K$ and F_π^{em} form factors
- qualitatively, the predicted form factors are predominantly "soft",
- the pion distribution amplitude $\varphi_\pi(u, 1\text{ GeV})$ close to asymptotic

  seemingly also in AdS/QCD , see [H.Grigoryan, A.Radyushkin (2008)]

Instead of Conclusion

"Were an analytic solution of QCD found, the contents of this review would become instantly obsolete"

M. Shifman , "Quark-Hadron Duality",
in "Handbook of QCD", (2001)

Congratulations to Misha !